EVERWIN MATRIC. HR. SEC. SCHOOL

15.07.2019 Special Test-Maths Time: 2½ hrs

STD: XII (A,B) Marks: 100

I. Choose the correct answer: 25x1=25

1. If $|adj\ (adj\ A)|=|A|^9$, then the order of the square matrix A is _____

1) 3 2) 4 3) 2 4) 5

2. $A=\begin{pmatrix}7 & 3\\ 4 & 2\end{pmatrix}$, then $9I_2-A=$

1) A^{-1} 2) $\frac{A^{-1}}{2}$ 3) $3A^{-1}$ d) $2A^{-1}$

3. If A^TA^{-1} is symmetric, then $A^2=$

1) A^{-1} 2) $(A^T)^2$ 3) A^T 4) $(A^{-1})^T$

4. If $A=\begin{pmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{pmatrix}$ and $A^T=A^{-1}$ then the value of x is

1) $\frac{-4}{5}$ 2) $\frac{-3}{5}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$

5. If $A=\begin{pmatrix}2 & 3\\ 5 & -2\end{pmatrix}$ be such that $\lambda A^{-1}=A$, then λ is _____.

1) 17 2) 14 3) 19 4) 21

6. If $A=\begin{pmatrix}3 & 5\\ 1 & 2\end{pmatrix}$, B=adj A and C=3A then $\frac{|adj\ B|}{|C|}$=_____

1) $\frac{1}{3}$ 2) $\frac{1}{9}$ 3) $\frac{1}{4}$ 4) 1

7. If $P(A)=P\left(\frac{A}{B}\right)$ then the system AX=B of linear equations is _____

1) consistent and has a unique solution

2) consistent and has infinitely many solution

3) consistent 4) inconsistent

8. The area of the triangle formed by the complex numbers z, iz and z+iz

1) $\frac{1}{2}|z|^2$ 2) $|z|^2$ 3) $\frac{3}{2}|z|$ 4) $2|z|^2$

9. If x is a non-zero complex number, such that $2iz^2=\bar{z}$ then $|z|$ is ___

1) $\frac{1}{2}$ 2) 1 3) 2 4) 3

10. If $|z|=1$, then the value of $\frac{1+z}{1+z}$ is _____

1) z 2) $\bar{z}$ 3) $\frac{1}{z}$ 4) 1

11. The principal argument of $(\sin 40^0 + i \cos 40^0)$ is

1) -110^0 2) -70^0 3) 70^0 4) 110^0

12. The product of all four values of $\left(cos\frac{\pi}{3}+i\ sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is _____

1) -2 2) -1 3) 1 4) 2

13. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}$=3k, then k is equal to _____.

1) 1 2) -1 3) $\sqrt{3}$i 4) $-\sqrt{3}$i

14. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is _____

1) cis $\frac{2\pi}{3}$ 2) cis $\frac{4\pi}{3}$ 3) -cis$\frac{2\pi}{3}$ 4) -cis$\frac{4\pi}{3}$

15. (1+i) (1+2i) (1+3i).....(1+ni)=x+iy, 2.5.10.....$(1+n^2)$ is _____

1) 1 2) i 3) x^2+y^2 4) $1+n^2$

16. If α and β are the roots then $x^2+x+1=0$,then $\alpha^{2020}+\beta^{2020}$ is _____

1) -2 2) -1 3) 1 4) 2

17. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7=A+B\omega$, then (A, B)=

1) (1, 0) 2) (-1, 1) 3) (0,1) 4) (1,1)

18. If z=x+iy is a complex number such that $|z+2|=|z-2|$, then the locus of z is _____.

1) real axis 2) imaginary axis 3) ellipse 4) circle

19. If $P=\begin{pmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{pmatrix}$ is the adjoint of 3x3 matrix A and $|A|=4$, then x

is

1) 15 2) 12 3) 14 4) 11

20. If A=$\begin{pmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{pmatrix}$ then adj(adj A) is _____

1) $\begin{pmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{pmatrix}$ 2) $\begin{pmatrix}6 & -6 & 8\\ 4 & -6 & 8\\ 0 & -2 & 2\end{pmatrix}$ 3) $\begin{pmatrix}-3 & 3 & -4\\ -2 & 3 & -4\\ 0 & 1 & -1\end{pmatrix}$ 4) $\begin{pmatrix}3 & -3 & 4\\ 0 & -1 & 1\\ 2 & -3 & 4\end{pmatrix}$

21. Matrices A and B will be inverse of each other only if _____

1) AB=BA 2) AB=BA=0 3) AB=0, BA=I 4) AB=BA=I

22. Given A=$\begin{pmatrix}1 & -2\\ -5 & 7\end{pmatrix}$ then A(adj A)=

1) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ 2) $\frac{1}{17}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ 3) $\frac{1}{3}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ 4) $\frac{1}{-3}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

23. Given P(A,B)=P(A) <number of unknowns, then the system has _____.

1) unique solution 2) no solution 3) 3 solutions

4) infinitely many solution

24. $\left(\frac{1-i}{1+i}\right)^{106}$=a+ib then (a,b) is _____.

1) (2, -1) 2) (1, 0) 3) (0,1) 4) (-1, 2)

25. If $-\bar{z}$ lies in the third quadrant, then z lies in the _____.

1) first quadrant 2) second quadrant 3) third quadrant

4) fourth quadrant

II. Answer for any 7 of the following: (Q.No.35 is compulsory) 7x2=14

26. If A=$\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ is non-singular, find A^{-1}.

27. Find adj(adj A) if adjA=$\begin{pmatrix}1 & 0 & 1\\ 0 & 2 & 0\\ -1 & 0 & 1\end{pmatrix}$

28. Find the rank by minor method: $\begin{pmatrix}1 & -2 & 3\\ 2 & 4 & -6\\ 5 & 1 & -1\end{pmatrix}$

29. Find the value of $i\ i^2\ i^3 \ldots\ldots i^{2000}$.

30. If z=x+iy, find Im(3z+4$\bar{z}$-4i) in rectangular form.

31. Which one of the points 10-8i, 11+6i is closest to 1+i.

32. If z=x+iy is a complex number such that $\left|\frac{z-4i}{z+4i}\right|$=1, show that the locus of z is real axis.

33. Find the inverse of $\begin{pmatrix}2 & -1\\ 5 & -2\end{pmatrix}$ by Gauss-Jordan method.

34. Verify A(adj A)=(adj A)A, for A=$\begin{pmatrix}8 & -4\\ -5 & 3\end{pmatrix}$

35. Find the square root of -7+24i

III. Answer for any 7 of the following: (Q.No.45 is compulsory)7x3=21

36. If A=$\begin{pmatrix}1 & \tan x\\ -\tan x & 1\end{pmatrix}$ show that $A^T\ A^{-1}$=$\begin{pmatrix}cos2x & -sin2x\\ sin2x & cos2x\end{pmatrix}$.

37. Find the rank of $\begin{pmatrix}1 & 1 & 1 & 3\\ 2 & -1 & 3 & 4\\ 5 & -1 & 7 & 11\end{pmatrix}$ by row reduction method.

38. Solve by Cramer's rule: 5x-2y+16=0;

x+3y-7=0

39. solve by matrix inversion method: 2x-y=8

3x+2y=-2

40. Find the value of the real numbers x and y, if the complex number (2+i)x+(1-i)y+2i-3 and x+(-1+2i)y+1+i are equal.

41. Show that $\left(2+i\sqrt{3}\right)^{10}$ -$\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

42. Show that the equation $z^3+2\bar{z}=0$ has 5 solutions.

43. Obtain the cartesian equation for the locus of z=x+iy in $|z-4|^2-|z-1|^2$=16.

44. If $(x_1+iy_1)\ (x_2+iy_2)\ldots\ldots\ldots (x_n+iy_n)$=a+ib show that

i) $(x_1^2+y_1^2)\ (x_2^2+y_2^2)\ \ldots\ldots\ldots (x_n^2+y_n^2)$=$a^2+b^2$

ii) $\sum_{r=1}^{n} tan^{-1}\left(\frac{y_r}{x_r}\right)$ =$\tan^{-1}\frac{b}{a}$ +2kπ, ktz.

45. Solve: $x^4+4=0$ (by De Moivre's theorem)

IV. Answer the following: 8x5=40

46. a) If $A=\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ show that $A^2-3A-7I_2=O_2$ Hence find A^{-1}.

(or)

b) If $z=(x+iy)$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$

47. a) Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$

(or)

b) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of 40% acid solution? (Use Cramer's rule to solve the problem)

48. a) Solve by Gaussian elimination method:

$2x-2y+3z=2$; $x+2y-z=3$, $3x-y+2z=1$

(or)

b) Solve the following homogeneous equations:

$2x+3y-z=0$; $x-y-2z=0$, $3x+y+3z=0$

49. a) If $x+\frac{1}{x}=2\cos\alpha$ and $y+\frac{1}{y}=2\cos\beta$ then show that,

i) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$

ii) $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$

(or)

b) If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in rectangular form.

50. a) If z_1, z_2 and z_3 are three complex numbers such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|az_1z_2+4z_1z_3+z_2z_3|=6$

(or)

b) By using Gaussian elimination method, balance the chemical reaction equation $C_2H_6+O_2\rightarrow H_2O+CO_2$

51. a) A boy is walking along the path $y=ax^2+bx+c$ through the points (-6,8) (-2,-12) and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian elimination method)

(or)

b) If $\left|Z-\frac{2}{z}\right|=2$, show that the greatest and least value of $|z|$ are $\sqrt{3}+1$ and $\sqrt{3}-1$ respectively.

52. a) If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ show that

i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$

ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

(or)

b) Verify $(AB)^{-1}=B^{-1}A^{-1}$ for $A=\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ and $B=\begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix}$

53u. a) For what value of μ the equations $x+y+3z=0$, $4x+3y+\mu z=0$, $2x+y+2z=0$ have a i) trivial solution ii) non-trivial solution.

(or)

b) If $\mathrm{Im}\left(\frac{2z+1}{iz+1}\right)$ is -2, then show that the locus of the point representing z in the argand plane is a straight line.